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DISPLACEMENT OF CURRENT IN THE BARS OF A CAGE OF AN INDUCTION M--ETC(U)

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August 1980

**DISPLACEMENT OF CURRENT IN
THE BARS OF A CAGE OF AN
INDUCTION MOTOR**

(Wypieranie Prądu w Pretach Klatki
Silnika Indukcyjnego),

by

A. Glowacki

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Procurement Executive, Ministry of Defence
Farnborough, Hants

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DISPLACEMENT OF CURRENT IN THE BARS OF A CAGE OF AN INDUCTION MOTOR

(WYPIERANIE PRĄDU W PRĘTACH KLATKI SILNIKA INDUKCYJNEGO)

by

A. Glowacki

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Translator

R. Wolski

Translation editor

L.G. Atkinson

EDITOR'S SUMMARY

The paper presents three methods for the calculation of the impedance of 'skin effect' bars of cage type induction machines.

(1) Using the effective penetration depth method presented by Liwschitz-Garik⁵.

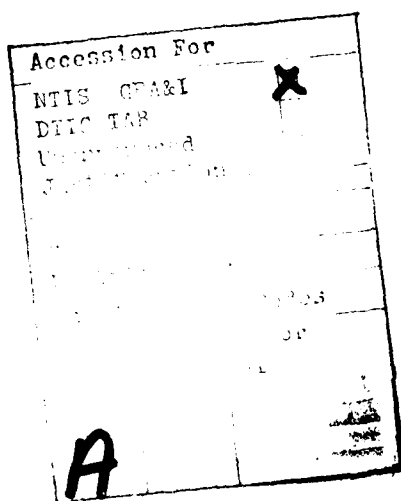
(2) By the use of a transmission line comprising a series of elemental networks as a model of the bar.

(3) By representing the bar as a network comprising a few sections, each section being related to a discreet portion of the bar.

Methods (2) and (3) above are similar to those presented by Babb and Williams⁴ and are also applicable to the analysis or design of multicage rotors.

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I INTRODUCTION

The above investigation was carried out for the following reasons: absence of a general method of calculation of displacement current in complex cross-section cages and inaccuracies in calculating some types of bars, e.g. dual-cage. A single cage deep-grooved motor can be considered as a special case of a dual-cage machine in which the throat between the cages has the same width as the neighbouring bar sections. Calculating such a bar by a method used for dual-cage motors will give results different from calculations based on a single-cage method. This points to an inconsistency of approach in the two methods which from a physical point of view has no justification.

To illustrate the mechanism of the displacement current a model of a groove in a ferromagnetic core can be used with three bars placed at different depths and short-circuited at both ends of the core (Fig 1). The currents I_1, I_2, I_3 flowing in the bars produce a magnetic field and if the core material permeability is very high ($\mu_r = \infty$) the field lines in the groove will be perpendicular to the sides. In the case illustrated in Fig 1 they will be straight if the distortions at the groove top and bottom are ignored. If the groove sides are inclined and particularly if they are not flat, the magnetic field becomes complex and calculations difficult.

For alternating current in the bars the described form of flux will induce voltages U_1, U_2, U_3 (Fig 1). The currents due to these voltages flow in the direction opposite to the flux-inducing currents giving different values of superimposed currents in particular bars; minimum in bar 1 and maximum in bar 3. The model shown in Fig 1 is particularly useful in numerical calculations of displacement current.

With non-uniform current density distribution the energy losses in the system are higher and the magnetic field energy is reduced. This can be interpreted as an increase in resistance and a reduction in permeance in comparison with direct current conditions.

The analytical solution of the problem has a long history and electrical machines text books include formulae for rectangular and trapezoid bars; calculations of different cross-sections present considerable difficulties. Satisfactory results have been achieved in recent times using a numerical method and treating the bar as a network of elementary conductors. A bar equivalent circuit method is also of interest and has been used so far for double-cage motor calculations. The method can be developed for use on any chosen bar complexity provided that the bar can be divided into n straight segments.

2 ANALYTICAL METHODS

These methods are based on solving Maxwell equations for a particular case of a bar placed in a ferromagnetic groove. The simplest is the case of a rectangular bar and for a large height to width ratio the problem can be solved as a single dimension system. Recently solutions have been provided for rectangular, trapezoid and circular sections in two-dimension fields⁹ but they are very complex and difficult to use in technical calculations.

'Classical' method

In practice the following definitions of the displacement current coefficient are used:

- (1) Coefficient for resistance k_r equal to the ratio of the bar alternating current resistance R_z and direct current resistance R_0

$$k_r = \frac{R_z}{R_0} \quad (1)$$

- (2) Coefficient for reactance k_x equal to the ratio of the bar ac reactance X_z and reactance calculated for dc conductance X_0

$$k_x = \frac{X_z}{X_0} = \frac{\lambda_z}{\lambda_0} \quad (2)$$

where λ is the permeance coefficient and $X = \omega \mu_0 \lambda$ with $\mu_0 = 4\pi 10^{-7} \text{ H/m}$.

For a rectangular bar a height coefficient, also known as 'bar reduced height' is calculated

$$\xi = h\alpha$$

where h is the actual bar height (mm), and α is calculated from the following equation:

$$\alpha = \sqrt{\frac{b_b}{b_s} \frac{\omega}{2} \mu \gamma} \quad (3)$$

b_b = bar width, b_s = slot width.

In cast cages groove and bar dimensions are identical and the ratio b_b/b_s is taken as equal to 1. In general bars are produced from non-magnetic material for which $\mu = \mu_0$. The specific electrical conductivity of the material γ is taken in MS/m and frequency $\omega_2 = 2\pi f_1 s$ (s = slip; f_1 = stator current frequency). Equation (3) can therefore be written as

$$\alpha = 2\pi \sqrt{f_1 s \gamma 10^{-7}} \quad (4)$$

or alternatively as

$$\alpha = 1.987 \times 10^{-3} \sqrt{s f_1 \gamma} \quad (5)$$

Displacement current coefficients for a rectangular bar are represented by

$$k_r = \phi(\xi) = \xi \frac{0.5(e^{2\xi} - e^{-2\xi}) + \sin 2\xi}{0.5(e^{2\xi} + e^{-2\xi}) - \cos 2\xi} \quad (6)$$

$$k_x = \psi(\xi) = \frac{2}{3\xi} \frac{0.5(e^{2\xi} - e^{-2\xi}) - \sin 2\xi}{0.5(e^{2\xi} + e^{-2\xi}) - \cos 2\xi} \quad (7)$$

The above relationships are shown graphically in Fig 2. For trapezoidal grooves it is necessary to allow for the effect of the bar sides tapering by use of the ratio $\epsilon = b_2/b_1$. This makes the formula more complicated and in practice graphs are used as in Refs 2 or 6. These graphs have been produced for straight flux lines and the error resulting from the simplification $2 > \epsilon > 0.5$ is negligible. The displacement current coefficient for such cases can be calculated relatively simply using an equivalent bar method.

The equivalent bar method

The current density distribution in a rectangular bar cross-section is approximately exponential due to the displacement current. An actual rectangular bar with non-uniform current distribution can be represented by an assumed bar with uniform current density distribution. Such a bar will have the same width but its height h_{ir} is chosen to make its current losses equal to the losses in the real bar; i.e. the assumed bar direct current resistance is equal to the real bar alternating current resistance. It follows therefore that

$$R_z = R_0(\xi) \quad (8)$$

and an appropriate value h_{ir} can be calculated.

Similarly height h_{ix} can be determined for the equivalent bar which has its direct current permeance equal to the real bar alternating current permeance

$$X_z = X_{0\psi}(\xi) . \quad (9)$$

Direct current resistances of the equivalent and real conductor will be inversely proportional to their cross-sections, i.e.

$$R_z = R_0 \frac{S_0}{S_z} = R_0 \frac{bh}{bh_{ir}} = R_0 \frac{h}{h_{ir}} . \quad (10)$$

Using equation (8) we obtain

$$h_{ir} = \frac{h}{\phi(\xi)} . \quad (11)$$

For reactances we have (allowing that for a rectangle $\lambda = h/3b$)

$$X_z = X_0 \frac{h_{ix}/3b}{h/3b} = X_0 \frac{h_{ix}}{h} . \quad (12)$$

In the above equation h_{ix} is the 'penetration depth' calculated with regard to the reactance

$$h_{ix} = h\psi(\xi) . \quad (13)$$

Applying the described method to a trapezoidal bar its total cross-section,

$$S_0 = 0.5h(b_1 + b_2) \quad (14)$$

determining the direct current resistance, is first calculated.

The equivalent bar cross-section

$$S_z = 0.5h_{ir}(b_1 + b_{3r}) . \quad (15)$$

The dimension b_{3r} depends on the penetration depth h_{ir} . From geometric relationship we calculate

$$b_{3r} = \frac{b_2 h + b_1 h_{ir} - b_2 h_{ir}}{h} . \quad (16)$$

Introducing the relationship $h_{ir} = h/\phi(\xi)$ we obtain

$$b_{3r} = \frac{b_2 \phi(\xi) + b_1 - b_2}{\phi(\xi)} . \quad (17)$$

and finally since $k_r = S_0/S_z$

$$k_r = \frac{(b_1 + b_2)\phi(\xi)}{b_2[2\phi(\xi) - 1] + b_1} \phi(\xi) \quad (18)$$

for $b_1 = b_2$ relationships for a rectangle are obtained.

An appropriate formula for k_x can be derived by calculating the ratio λ_z/λ_0 for two trapezoids of same apex angle β : one with base b_{3x} and the other with base b_1 (Fig 3). Width b_{3x} depends on $\psi(\xi)$ from the relationship

$$b_{3x} = (b_1 - b_2)\psi(\xi) + b_2 .$$

Calculating k_x for a trapezoid use is made of an appropriate formula for permeance λ . Making

$$\epsilon_1 = \frac{b_2}{b_1} \quad \text{and} \quad \epsilon_2 = \frac{b_2}{b_{3x}}$$

then the following relationship is obtained

$$k_x = \frac{\epsilon_1 - 1}{\epsilon_2 - 1} \frac{\frac{\epsilon_2^2 - 3}{4} + \frac{\ln \epsilon_2}{\epsilon_2 - 1}}{\frac{\epsilon_1^2 - 3}{4} + \frac{\ln \epsilon_1}{\epsilon_1 - 1}} . \quad (19)$$

Similar relationships can be derived for different bar shapes. It appears that the penetration depth for every shape is the same as for an equivalent rectangular bar of the same total height. All four bar shapes shown in Fig 3 have therefore the same penetration depth: h_{ir} described by equation (11) and h_{ix} by equation (13). It can also be said that the displacement current coefficient $k_r = R_z/R_0$ for each bar is equal to the ratio of the total bar area to the shaded area in Fig 3 (dependent on h_{ir}). Similarly the

coefficient k_x is equal to the permeance ratio of the shaded area (dependent on h_{ix}) to the total area. The calculation of the displacement current coefficient k_r is reduced to a comparison of the two areas, and coefficient k_x is obtained from a comparison of the two permeance values calculated for direct current. Calculation of permeance can sometimes be difficult.

Method of elementary conductors network

A solid bar in a groove can be divided into thin strips along the lines of field without changing the effects taking place in the bar. It can be assumed that strips are so thin that there is no displacement current in them (i.e. their current density distribution is uniform). The strips are insulated from each other by a negligible insulation thickness and are connected at both sides of the core forming a network of elementary conductors. Each conductor is characterised by its cross-section area S_i , median line length between groove walls c_i and conductivity of the conductor material γ_c .

If the neighbouring field lines are parallel then the permeance coefficient of i th element can be calculated from

$$\lambda_i = \frac{S_i}{c_i} \quad (20)$$

The unit resistance (calculated for unit bar length) of i th element is given by

$$r_i = \frac{1}{\gamma_i S_i} \quad (21)$$

The inductance of the elements is calculated from

$$L_{m,k} = \frac{\sum_{i=\max(m,k)}^{i=n} \Phi}{I} = \frac{I \mu_0 \sum_{i=\max(m,k)}^{i=n} \lambda_i}{I} = \mu_0 \sum_{i=\max(m,k)}^{i=n} \lambda_i \quad (22)$$

The coupling system between the elements is similar to that shown in Fig 1. To calculate the inductance $L_{m,k}$ a sum λ_i is taken starting with the larger from the values of m or k ; this corresponds to the flux coupling (containing) the elements m and k . The elements are numbered starting from

the groove bottom up. If $m = k$ the relationship gives the value of the self-inductance. For $m \neq k$ we obtain mutual inductance resulting from the magnetic coupling of elements m and k .

For a system of n elementary conductors the following n equations can be written

$$\begin{aligned} V &= I_1(r_1 + j\omega L_{11}) + I_2 j\omega L_{12} + \dots + I_n j\omega L_{1n} \\ V &= I_1 j\omega L_{21} + I_2(r_2 + j\omega L_{22}) + \dots + I_n j\omega L_{2n} \\ &\dots \dots \dots \\ V &= I_1 j\omega L_{n1} + I_2 j\omega L_{n2} + \dots + I_n(r_n + j\omega L_{nn}) \end{aligned} \quad (23)$$

and

$$I_t = I_1 + I_2 + I_3 + \dots + I_n \quad (24)$$

These equations can be transformed (using equation (22))

$$\begin{aligned} V &= I_1 r_1 + j\omega \mu_0 \left[\lambda_1 I_1 + \lambda_2 (I_1 + I_2) + \dots + \lambda_n \sum_{i=1}^n I_i \right] \\ V &= I_2 r_2 + j\omega \mu_0 \left[\lambda_2 (I_1 + I_2) + \lambda_3 (I_1 + I_2 + I_3) + \dots + \lambda_n \sum_{i=1}^n I_i \right] \\ &\dots \dots \dots \\ V &= I_n r_n + j\omega \mu_0 \lambda_n \sum_{i=1}^n I_i \end{aligned} \quad (25)$$

The equivalent circuit shown in Fig 4 corresponds to the system of equations (25). Inductance calculation using formula (22) leads to a small error since it is assumed that the flux produced by the i th element is contained completely within the element. However, the flux produced by the i th element current is closed above it and therefore to calculate the permeance λ_i an upper line length of the element is more appropriate than its middle line. Klokov⁸ suggests a double system of elementary conductors mutually displaced and one being used for the calculation of permeance and the other for resistance.

Formation of pairs of elementary conductors appears to be more practical. To calculate λ_i a pair 'i' and 'i + 1' is taken and for calculating r_i the pair 'i' and 'i - 1'. In this case equations (20) and (21) take the following form:

$$r_i = \frac{1}{\gamma_i (S_i + S_{i-1})} . \quad (26)$$

$$\lambda_i = \frac{S_i + S_{i+1}}{c_i^2} . \quad (27)$$

The value of i is changed by 2, *ie* the elements (connected as pairs) are numbered $i = 2, 4, 6, \dots, n$, counting from the bar bottom. The zone $S_{i+1} = 0$ and n must be even. In consequence also the form of the general equations is changed. For the elements connected as pairs the currents I_2, I_4, I_6 , etc will occur. The $n/2$ set of equations can be solved by different methods including matrix calculus. It is more convenient, by side subtraction of equations, to transform the set into the following general form

$$I_{(i+2)} = \frac{r_i}{r_{(i+2)}} I_i + j\omega\mu_0 \frac{\lambda_i}{r_{(i+2)}} \sum_2^i I_i . \quad (28)$$

In the obtained sequence of equations the first current, *ie* I_2 , remains unknown. It can be assumed initially that $I_2 = 1$ with all other currents then referred to I_2 . If all branch currents in the network are known the total current I_t is calculated as their sum. This current can be calculated as a normalized quantity (in amperes) using an equivalent circuit for the whole machine (for a given load or slip). However, for the calculation of the displacement current coefficient or impedances this is not necessary. If the current I_t is known (I_t and all branch currents are referred to I_2) it is possible to change the scale of the currents by referring them to I_t . It is sufficient to make the scale change only for one conductor. For convenience calculations are carried out separately on real and imaginary parts denoted A and B respectively.

In accordance with convention the current in the first conductor numbered $i = 2$ is equal to 1, *ie* $A_2 = 1$, $B_2 = 0$ and $I_2 = 1 + j0$. For a conductor numbered $i + 2$ (*ie* 4, 6, 8, ..., n)

$$A_{(i+2)} = \frac{r_i}{r_{(i+2)}} A_i - \omega \mu_0 \frac{\lambda_i}{r_{(i+2)}} \sum_2^n B_i . \quad (29)$$

$$B_{(i+2)} = \frac{r_i}{r_{(i+2)}} B_i + \omega \mu_0 \frac{\lambda_1}{r_{(i+2)}} \sum_2^n A_i . \quad (30)$$

Scale change is then made taking as reference current $I_t = A_t + jB_t$. This is done only for nth conductor.

The components of current I'_n , reduced to I_t will be equal to:

$$A'_n = \frac{A_n A_t + B_n B_t}{A_t^2 + B_t^2} . \quad (31)$$

$$B'_n = \frac{B_n A_t - A_n B_t}{A_t^2 + B_t^2} . \quad (32)$$

If R_t is an equivalent resistance of the whole circuit and X_t its reactance, R and X are calculated from the circuit equation

$$V = I_t (R_t + jX_t)$$

and from type (28) equation for the nth conductor. In this equation the sum currents I_t is equal to I_t and therefore

$$R_t = A'_n r_n \quad (33)$$

$$X_t = B'_n r_n + \omega \mu_0 \lambda_n . \quad (34)$$

Reactance X_0 is calculated from equation

$$X_0 = \omega \mu_0 \lambda_0 . \quad (35)$$

The permeance coefficient λ_0 can be calculated from the system of elementary conductors for low frequency (as a limiting value for $\omega \rightarrow 0$) or by a different method, eg from the magnetic field energy in the slot.

Resistance R_0 is calculated from

$$R_0 = \frac{1}{\sum_{i=1}^n S_i \gamma_i} \quad (36)$$

and displacement current coefficient from

$$k_r = \frac{R_t}{R_0} \quad (37)$$

$$k_x = \frac{X_t}{X_0} \quad (38)$$

The elementary conductor method can be used to calculate any slot described by data tables of i , S_t , c_t , γ_t . The calculation can be developed further to include current density distribution and losses in bar cross-section. Accuracy of calculation depends on the number of the elementary conductors used and on satisfying the requirement of division only along the lines of field. Depending on groove shape n is taken to be from 50 to 150. Definition of the line diagram of the slot magnetic field and the preparation of the data table present considerable difficulties. Existing methods of field distribution calculation are too complex to be practicable. Ref 10 gives a method of approximate reproduction of slot field by division into segments of typical field diagram.

Equivalent circuits with lumped parameters

The unit of calculation is in this case a segment, *ie* a simple shaped (rectangular, trapezoid or circular) part of the slot as in Fig 5.

The permeance of each segment can be calculated, taking into account the actual field diagram, by for example conformal mapping. Segments containing current carrying conductors are characterised by three permeance coefficients: λ' - calculated for current carrying segments, λ'' - calculated for non-current carrying segments, $\lambda^{(m)}$ - representing the coupling of the segment with other segments. In the case of a segment not containing current carrying conductors λ' and $\lambda^{(m)}$ are equal to zero, as are the coefficients λ^m and λ'' for the segment at the bottom of the slot. Refs 7 and 11 include a method of permeance coefficient calculation.

Segments can be considered to be separate bars separated from each other in the groove by a very thin insulating layer and connected by short-circuiting rings at both ends of the core. A cross-section of a segment should be sufficiently small so that its current density distribution could be considered as uniform. If by a natural division of a bar the above condition is not satisfied then an additional division of larger segments will be necessary. The top parts of the bar exhibit largest current density variations. It can be said that a set of voltage equations for circuits of particular segments is represented by an equivalent circuit in Fig 6. The circuit as shown applies to four segments but it can be easily developed for any larger number. In general, it is not necessary to divide the bar into more than seven segments. The task involves calculating the equivalent circuit resistance R_t and reactance X_t for a given frequency ω . The solution of the circuit begins with the segments numbered 1 and 2, ie from the groove bottom up. Intermediate results are useful and will be included. Branches 1 and 2 will be substituted by one of the following form

$$Z_{t1} = R_{t1} + jX_{t1}$$

in which

$$R_{t1} = \frac{R_1 R_2 (R_1 + R_2) + X_1^2 R_2 + X_2^2 R_1}{(R_1 + R_2)^2 + (X_1 + X_2)^2} \quad (39)$$

$$X_{t1} = \frac{X_1 X_2 (X_1 + X_2) + R_1^2 X_2 + R_2^2 X_1}{(R_1 + R_2)^2 + (X_1 + X_2)^2} + X_{2m} \quad (40)$$

Branch Z_{t1} is joined with branch 3 giving Z_{t2} with elements

$$R_{t2} = \frac{R_{t1} R_3 (R_{t1} + R_3) + X_{t1}^2 R_3 + X_3^2 R_{t1}}{(R_{t1} + R_3)^2 + (X_{t1} + X_3)^2} \quad (41)$$

$$X_{t2} = \frac{X_{t1} X_3 (X_{t1} + X_3) + R_{t1}^2 X_3 + R_3^2 X_{t1}}{(R_{t1} + R_3)^2 + (X_{t1} + X_3)^2} + X_{3m} \quad (42)$$

Continuing similarly we obtain

$$R_{t3} = \frac{R_{t2}R_4(R_{t2} + R_4) + X_{t2}^2R_4 + X_4^2R_{t2}}{(R_{t2} + R_4)^2 + (X_{t2} + X_4)^2} . \quad (43)$$

$$X_{t3} = \frac{X_{t2}X_4(X_{t2} + X_4) + R_{t2}^2X_4 + R_4^2X_{t2}}{(R_{t2} + R_4)^2 + (X_{t2} + X_4)^2} + X_{4m} \quad (44)$$

etc.

In order to calculate the displacement current coefficient it is necessary, in accordance with equations (37) and (38), to calculate values R_0 and X_0 . They are obtained from equations (36) and (35) as the limit when $\omega = 2\pi f \rightarrow 0$. Substituting

$$R_1 = \frac{1}{\gamma S_1}, \quad R_2 = \frac{1}{\gamma S_2}$$

etc. (S_1, S_2 - cross-section area of segments 1, 2, etc) we obtain

$$R_{01} = \frac{1}{\gamma(S_1 + S_2)} \quad (45)$$

$$X_{01} = \frac{S_2^2X_2 + S_1^2X_1}{(S_1 + S_2)^2} + X_{2m} \quad (46)$$

$$R_{02} = \frac{1}{\gamma(S_1 + S_2 + S_3)} . \quad (47)$$

$$X_{02} = \frac{S_3^2X_3 + (S_1 + S_2)^2X_{01}}{(S_1 + S_2 + S_3)^2} + X_{3m} . \quad (48)$$

$$R_{03} = \frac{1}{\gamma(S_1 + S_2 + S_3 + S_4)} . \quad (49)$$

$$X_{03} = \frac{S_4^2X_4 + (S_1 + S_2 + S_3)^2X_{02}}{(S_1 + S_2 + S_3 + S_4)^2} + X_{4m} . \quad (50)$$

In the case of a smaller number of segments, eg 3 the calculation ends after $R_{t2}, X_{t2}, R_{02}, X_{02}$.

Reactances depending on the values of the permeance coefficients of the segments are calculated from

$$X_1 = c_x (\lambda_1' + \lambda_2'' - \lambda_2^{(m)}) \quad . \quad (51)$$

$$X_2 = c_x (\lambda_2' - \lambda_2^{(m)}) \quad . \quad (52)$$

$$X_3 = c_x (\lambda_3' - \lambda_3^{(m)}) \quad . \quad (53)$$

$$X_4 = c_x (\lambda_4' - \lambda_4^{(m)}) \quad . \quad (54)$$

.....

$$X_{2m} = c_x (\lambda_3'' + \lambda_2^{(m)} - \lambda_3^{(m)}) \quad . \quad (55)$$

$$X_{3m} = c_x (\lambda_4'' + \lambda_3^{(m)} - \lambda_4^{(m)}) \quad . \quad (56)$$

$$X_{4m} = c_x \lambda_4^{(m)} \quad . \quad (57)$$

It is easy to see the construction of the formulae. In three segments calculation we take formulae for $X_1, X_2, X_3, X_{2m}, X_{3m}$; X_{3m} will be equal to $c_x \lambda_4^{(m)}$ because the permeance coefficients for higher numbered segments are equal to zero.

It is now necessary to define the units. It is more convenient to work in $\mu\Omega$ with γ being in MS/m. The constant $c_x = \omega\mu_0 = 0.8\pi^2 f = 7.896f$. For 50 Hz, $c_x = 394.78$. The calculation is carried out for a unit length of the bar (1 m). Having calculated R_t and X_t for the rotor circuit, these values are then transferred to the stator circuit and the equivalent circuit is solved for the whole motor.

3 CONCLUSIONS

The three methods presented here of approximate calculation of displacement current in the bars of a cage make it possible to solve almost all the problems occurring in practice. The elementary conductor method is the most general but the high effort involved in the calculations demands the use of computers. The method of segmentation requires much less effort since each

segment 'replaces' 10-20 elementary conductors. As an example the following calculations were carried out for a trapezoidal bar with bases $b_1 = 10$, $b_2 = 5$, height $h = 32$ mm. Conductivity $\gamma = 23$ MS/m. The method of elementary conductors with circular arc division into n equal height layers produced the following results.

$n = 12$	$k_r = 2.41$	$k_x = 0.74$
24	2.48	0.73
48	2.5	0.73

In calculations using the segments method the bar was first divided into two equal height parts and then the upper part was similarly divided again. The results obtained were

$n = 2$	$k_r = 2.15$	$k_x = 0.81$
3	2.43	0.75
4	2.5	0.74
5	2.51	0.74

With division into $n = 5$ segments the heights were equal to 16, 8, 4, 2, 2 mm. It can be seen that $n = 4$ already gives an adequately accurate result. The described method has particular significance in calculating double-cage motors since none of the previously used methods offered sufficiently general and accurate solutions.

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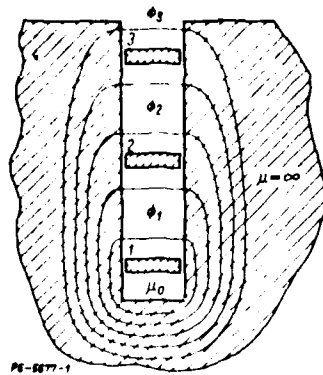


Fig 1 Model of a groove with bars illustrating the displacement current mechanism

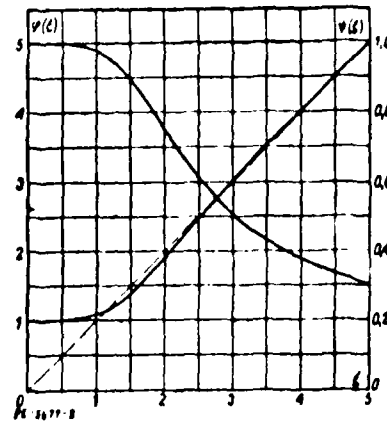


Fig 2 Graphs $\phi(\xi)$ and $\psi(\xi)$ for a rectangular bar

$$b_{2x} = (b_1 - b_2) \psi(\xi) + b_2$$

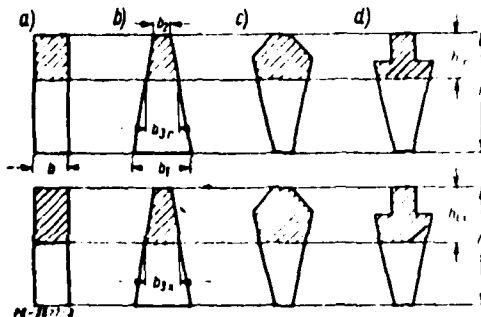


Fig 3 Bars of different shapes but same 'penetration depth': (a) equivalent rectangular cross-section bar; (b), (c), (d) cross-sections used in practice

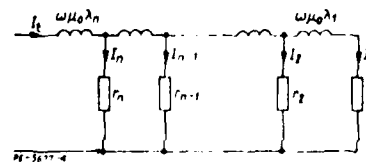


Fig 4 An equivalent circuit for a cage bar divided into n elementary conductors

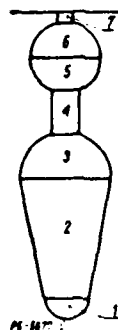


Fig 5 An example of a complex slot (bar) division into segments

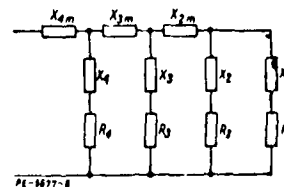


Fig 6 An equivalent circuit for a bar divided into four segments

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